

1 **Claims**

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3 1. An interactive method for demonstrating an interrelationship  
4 between different representations of a mathematical relationship, including the  
5 steps of:

6 (a) defining a mathematical equation;

7 (c) simultaneously displaying at least two of multiple representations of the  
8 defined mathematical equation, wherein the available types of multiple  
9 representations include a graphical representation in the form of a  
10 graph, a numerical representation in the form of a table of values, and a  
11 symbolic representation in the form of an equation expressed in terms of  
12 standard mathematical nomenclature, wherein one of the displayed  
13 representations is the graphical representation;

14 (c) manipulating the graphical representation; and

15 (d) processing the manipulation to substantially simultaneously and  
16 correspondingly update the other displayed representation of the  
17 mathematical relationship in accordance with the manipulation of the  
18 graphical representation,

19 whereby a user of the method is able to substantially immediately  
20 observe the effect of changes made to the graphical representation via  
21 its manipulation on the other of the at least two displayed  
22 representations.

23

24 2. A method as claimed in claim 1 wherein the step of defining a  
25 mathematical equation includes selecting a mathematical equation from a list of  
26 predefined mathematical equations.

27

28 3. A method as claimed in claim 2 wherein the list of predefined  
29 mathematical equations includes equations selected from one or more of:

30 (a) linear mathematical relations;

31 (b) polynomial mathematical relations;

32 (c) exponential mathematical relations;

33 (d) logarithmic mathematical relations;

- 1 (e) power mathematical relations;  
 2 (f) trigonometric mathematical relations; and  
 3 (g) conic section mathematical relations.  
 4

5 4. A method as claimed in claim 2 wherein the list of predefined  
 6 mathematical equations includes at least two equations selected from::

- 7 (a) a linear mathematical equation described by  $y = m(x - h) + k$ ;  
 8 (b) a quadratic mathematical equation described by  $y = a(x - h)^2 + k$ ;  
 9 (c) a circular mathematical equation described by  $(x - h)^2 + (y - k)^2 = r^2$ ;  
 10 (d) an elliptical mathematical equation described by  $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ ;  
 11 (e) a hyperbolic mathematical equation described by  $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ ;  
 12 (f) a hyperbolic mathematical equation described by  $\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$ ;  
 13 (g) a parabolic mathematical equation described by  $y = m(x - h)^2 + k$ ;  
 14 (h) a parabolic mathematical equation described by  $(y - k)^2 = c(x - h)$ ;  
 15 (i) a general exponential mathematical equation described by  $y = ba^x + k$ ;  
 16 (j) a natural exponential mathematical equation described by  $y = be^{ax} + k$ ;  
 17 (k) a logarithmic mathematical equation of the form  $y = b \ln(a(x - h)) + k$ ;  
 18 (l) a power mathematical equation described by  $y = a(x - h)^r + k$ ;  
 19 (m) a sine mathematical equation described by  $y = b \sin(a(x - h)) + k$ ; and  
 20 (n) a cosine mathematical equation described by  $y = b \cos(a(x - h)) + k$ ;

21 where x and y are variable parameters and a, b, m, h, k and r are parameters  
 22 according to standard mathematical nomenclature, the numerical values for  
 23 which included in a particular predefined mathematical relation are user  
 24 definable.  
 25

26 5. A method as claimed in claim 1 wherein manipulation  
 27 mechanisms available for manipulating the graphical representation of the  
 28 mathematical relation include:

- 1 (a) translating the graph with respect to a set of coordinate axes; and  
2 (b) dilating the graph with respect to a set of coordinate axes.

3 6. A method as claimed in claim 1 wherein the method is performed  
4 using a programmed computer in combination with a stylus device.

5  
6 7. An interactive method for demonstrating an interrelationship  
7 between representations of a mathematical relationship, one of which  
8 representations is a graphical representation of the relationship relative to co-  
9 ordinate axes, the method including:

- 10 (a) simultaneously displaying on a visual display a mathematical relation in  
11 the form of the graphical representation and in another format being  
12 either an algebraic formula or a tabulated set of data which describes  
13 the graph or both;  
14 (b) locating a stylus on the graphical representation on a position sensing  
15 screen associated with the visual display and a processor for the  
16 position sensing device to sense stylus position and provide stylus  
17 position data to the processor;  
18 (c) moving the stylus over the position sensing screen for the processor to  
19 process changing stylus position data and manipulate the displayed  
20 graphical representation to change its shape or position relative to the  
21 co-ordinate axes in accordance with the motion of the stylus over the  
22 position sensing screen;  
23 (d) wherein displayed information in said another format is substantially  
24 simultaneously and correspondingly changed to continually describe the  
25 graph as it is manipulated;  
26 whereby a user of the method is able to substantially immediately  
27 observe the effect of changes made to the graphical representation via  
28 its manipulation on said another of the displayed representations.

29  
30 8. Apparatus for interactively demonstrating an interrelationship  
31 between different representations of a mathematical relation, the  
32 apparatus including

1 a visual display device and a position sensitive touch screen  
2 associated with the visual display device, a processor operatively linked  
3 with the visual display device and the touch screen, and a memory for  
4 storing application software, data and visual display information for the  
5 processor, visual display device and touch screen;

6 the visual display device and position sensitive touch screen  
7 providing for definition by use of a stylus of a mathematical equation;

8 wherein the visual display device simultaneously displays at least  
9 two of multiple possible representations of the defined mathematical  
10 equation, the multiple possible representations including a graphical  
11 representation in the form of a graph, a numerical representation in the  
12 form of a table of relation values, and a symbolic representation in the  
13 form of an equation expressed in terms of standard mathematical  
14 nomenclature,

15 and wherein the graphical representation is selectable for display  
16 as one of said at least two displayed representations;

17 wherein a stylus is positionable on the touch screen on the  
18 graphical representation and movable over the touch screen to generate  
19 changing position data, the apparatus being responsive to said changing  
20 position data for the visual display device to display in real time a  
21 manipulation of the graphical representation corresponding to the  
22 movement of the stylus;

23 and wherein the apparatus is also responsive to said changing  
24 position data to substantially simultaneously and correspondingly update  
25 the other displayed representation of the mathematical relationship in  
26 accordance with the manipulation of the graphical representation;  
27 whereby a user is able to substantially immediately observe the effect of  
28 changes made to the graphical representation via its manipulation on the  
29 other of the at least two displayed representations.

30  
31 9. Apparatus as claimed in claim 8 wherein the visual display device  
32 and position sensitive touch screen provide for selection of a

1 mathematical equation from a list of predefined mathematical equations  
 2 stored in the memory whereby the mathematical equation is definable.

3  
 4 10. Apparatus as claimed in claim 9 wherein the list of  
 5 predefined mathematical equations stored in the memory includes equations  
 6 selected from one or more of:

- 7 (a) linear mathematical relations;  
 8 (b) polynomial mathematical relations;  
 9 (c) exponential mathematical relations;  
 10 (d) logarithmic mathematical relations;  
 11 (e) power mathematical relations;  
 12 (f) trigonometric mathematical relations; and  
 13 (g) conic section mathematical relations.

14  
 15 11. Apparatus as claimed in claim 9 wherein the list of predefined  
 16 mathematical equations stored in the memory includes at least two equations  
 17 selected from:

- 18 (a) a linear mathematical equation described by  $y = m(x - h) + k$ ;  
 19 (b) a quadratic mathematical equation described by  $y = a(x - h)^2 + k$ ;  
 20 (c) a circular mathematical equation described by  $(x - h)^2 + (y - k)^2 = r^2$ ;  
 21 (d) an elliptical mathematical equation described by  $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ ;  
 22 (e) a hyperbolic mathematical equation described by  $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ ;  
 23 (f) a hyperbolic mathematical equation described by  $\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$ ;  
 24 (g) a parabolic mathematical equation described by  $y = m(x - h)^2 + k$ ;  
 25 (h) a parabolic mathematical equation described by  $(y - k^2) = c(x - h)$ ;  
 26 (i) a general exponential mathematical equation described by  $y = ba^x + k$ ;  
 27 (j) a natural exponential mathematical equation described by  $y = be^{ax} + k$ ;  
 28 (k) a logarithmic mathematical equation of the form  $y = b \ln(a(x - h)) + k$ ;

- 1 (l) a power mathematical equation described by  $y = a(x - h)^r + k$  ;  
 2 (m) a sine mathematical equation described by  $y = b\sin(a(x - h)) + k$  ; and  
 3 (n) a cosine mathematical equation described by  $y = b\cos(a(x - h)) + k$  ;  
 4 where x and y are variable parameters and a, b, m, h, k and r are parameters  
 5 according to standard mathematical nomenclature, the numerical values for  
 6 which included in a particular predefined mathematical relation are user  
 7 definable.

- 8  
 9 12. Apparatus as claimed in claim 8 wherein the application software  
 10 provides manipulation mechanisms for manipulating the graphical  
 11 representation of the mathematical equation which include  
 12 (a) translating the graph with respect to a set of coordinate axis;  
 13 (b) dilating the graph with respect to a set of coordinate axis.

- 14  
 15 13. Apparatus as claimed in claim 8 which is a hand-held computer  
 16 device.

- 17  
 18 14. A hand held computer device for demonstrating an  
 19 interrelationship between different representations of a mathematical  
 20 relationship, including  
 21 (a) a visual display unit for displaying multiple representations of a  
 22 mathematical relationship, wherein the available representations include:  
 23 (i) a graphical representation,  
 24 (ii) a numerical representation in the form of tabulated data, and  
 25 (iii) a symbolic representation in the form of a mathematical equation;  
 26 (b) a memory for storing application software, data, and visual display  
 27 information;  
 28 (c) a stylus;  
 29 (d) a position sensing touch screen associated with the visual display unit;  
 30 and  
 31 (e) a processor coupled to the visual display unit, memory, and position  
 32 sensing device touch screen, for updating multiple representations of the

1 mathematical relationship according to a manipulation of a graphical  
2 representation displayed on the visual display unit, wherein the  
3 manipulation of the graphical representation occurs in response to a  
4 motion of the stylus on the position sensing touch screen;  
5 whereby a user of the device is able to substantially immediately  
6 observe the effect of changes made to the graphical representation via  
7 its manipulation on the other of the at least two displayed  
8 representations.